# 12

# Limits and Derivatives

## TOPIC 1 Limit of a Function, Left Hand & Right Hand limits, Existance of Limits, Sandwitch Theorem, Evaluation of Limits when X→ ∞, Limits by Factorisation, Substitution & Rationalisation

**1.** If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim_{x\to\alpha^+} \frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to: **[Sep. 05, 2020 (I)]**

(a) $\frac{3}{2}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

**2.** $\lim_{x\to 0} \frac{x\left(e^{(\sqrt{1+x^2+x^4}-1)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$ **[Sep. 05, 2020 (II)]**

(a) is equal to $\sqrt{e}$ (b) is equal to 1
(c) is equal to 0 (d) does not exist

**3.** Let $[t]$ denote the greatest integer $\le t$. If for some $\lambda \in \mathbf{R} - \{0, 1\}$, $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x+[x]}\right| = L$, then $L$ is equal to : **[Sep. 03, 2020 (I)]**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

**4.** If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$, then the value of $k$ is __________. **[NA Sep. 03, 2020 (I)]**

**5.** $\lim_{x\to 2} \frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to______. **[NA Jan. 7, 2020 (I)]**

**6.** Let $f(x) = 5-|x-2|$ and $g(x) = |x+1|$, $x \in$ R. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim_{x\to -\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to : **[April 12, 2019 (II)]**

(a) 1/2 (b) –3/2 (c) –1/2 (d) 3/2

**7.** $\lim_{x\to 0} \frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x - x+1}}$ is **[April 12, 2019 (II)]**

(a) 6 (b) 2 (c) 3 (d) 1

**8.** If $\lim_{x\to 1} \frac{x^4-1}{x-1} = \lim_{x\to k} \frac{x^3-k^3}{x^2-k^2}$, then k is: **[April 10, 2019 (I)]**

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

**9.** If $\lim_{x\to 1} \frac{x^2-ax+b}{x-1} = 5$, then $a+b$ is equal to : **[April 10, 2019 (II)]**

(a) –4 (b) 5 (c) –7 (d) 1

**10.** $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals : **[April 8, 2019 (I)]**

(a) $4\sqrt{2}$ (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) 4

**11.** $\lim_{x\to \frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is: **[Jan. 12, 2019 (I)]**

(a) 4 (b) $4\sqrt{2}$ (c) $8\sqrt{2}$ (d) 8

12. $\lim\limits_{x \to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$ is equal to: **[Jan. 12, 2019 (II)]**

(a) $\frac{1}{\sqrt{2\pi}}$ (b) $\sqrt{\frac{2}{\pi}}$ (c) $\sqrt{\frac{\pi}{2}}$ (d) $\sqrt{\pi}$

13. Let [x] denote the greatest integer less than or equal to x. Then :

$$\lim\limits_{x \to 0} \frac{\tan\left(\pi \sin^2 x\right) + \left(|x| - \sin\left(x[x]\right)\right)^2}{x^2}:$$

**[Jan. 11, 2019 (I)]**

(a) does not exist (b) equals $\pi$
(c) equals $\pi + 1$ (d) equals 0

14. $\lim\limits_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to : **[Jan. 11, 2019 (II)]**

(a) 0 (b) 2 (c) 4 (d) 1

15. For each t ∈ **R**, let [t] be the greatest integer less than or equal to t. Then, **[Jan. 10, 2019 (I)]**

$$\lim\limits_{x \to 1+} \frac{(1 - |x| + \sin|1-x|) \sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(a) equals 1 (b) equals 0
(c) equals – 1 (d) does not exist

16. $\lim\limits_{y \to 0} \frac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$ **[Jan. 9, 2019 (I)]**

(a) exists and equals $\frac{1}{4\sqrt{2}}$

(b) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(c) exists and equals $\frac{1}{2\sqrt{2}}$

(d) does not exist

17. For each $x \in$ **R**, let $[x]$ be greatest integer less than or equal to $x$. Then **[Jan. 09, 2019 (II)]**

$\lim\limits_{x \to 0} \frac{x\left([x] + |x|\right) \sin[x]}{x}$ is equal to:

(a) – sin 1 (b) 1 (c) sin 1 (d) 0

18. $\lim\limits_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ equals. **[Online April 15, 2018]**

(a) 1 (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

19. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals : **[2017]**

(a) $\frac{1}{4}$ (b) $\frac{1}{24}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

20. $\lim\limits_{x \to 3} \frac{\sqrt{3x} - 3}{\sqrt{2x-4} - \sqrt{2}}$ is equal to : **[Online April 8, 2017]**

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$

21. $\lim\limits_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to : **[2015]**

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) 3

22. $\lim\limits_{x \to 0} \frac{e^{x^2} - \cos x}{\sin^2 x}$ is equal to : **[Online April 10, 2015]**

(a) 2 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{5}{4}$

23. $\lim\limits_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to: **[2014]**

(a) $-\pi$ (b) $\pi$ (c) $\frac{\pi}{2}$ (d) 1

24. If $\lim\limits_{x \to 2} \frac{\tan(x-2)\left\{x^2 + (k-2)x - 2k\right\}}{x^2 - 4x + 4} = 5,$

then k is equal to: **[Online April 11, 2014]**

(a) 0 (b) 1 (c) 2 (d) 3

25. $\lim\limits_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to **[2013]**

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

26. $\lim\limits_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ equals **[Online May 26, 2012]**

(a) $-\pi$ (b) 1 (c) – 1 (d) $\pi$

27. $\lim\limits_{x \to 0} \left(\frac{x - \sin x}{x}\right) \sin\left(\frac{1}{x}\right)$ **[Online May 7, 2012]**

(a) equals 1 (b) equals 0
(c) does not exist (d) equals – 1

28. Let $f : R \to [0,\infty)$ be such that $\lim_{x\to 5} f(x)$ exists and

$$\lim_{x\to 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$$ **[2011RS]**

Then $\lim_{x\to 5} f(x)$ equals :

(a) 0 (b) 1 (c) 2 (d) 3

29. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ **[2011]**

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$

(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

30. Let f : R → R be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)} = 1$ then $\lim_{x\to\infty}\frac{f(2x)}{f(x)} =$ **[2010]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 1

31. Let $\alpha$ and $\beta$ be the distinct roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to **[2005]**

(a) $\frac{a^2}{2}(\alpha-\beta)^2$ (b) 0

(c) $\frac{-a^2}{2}(\alpha-\beta)^2$ (d) $\frac{1}{2}(\alpha-\beta)^2$

32. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is **[2003]**

(a) $\infty$ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

33. $\lim_{x\to 0}\frac{\log x^n-[x]}{[x]}, n\in N$, ([x] denotes greatest integer less than or equal to x) **[2002]**

(a) has value -1 (b) has value 0

(c) has value 1 (d) does not exist

34. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ **[2002]**

(a) $e^4$ (b) $e^2$ (c) $e^3$ (d) 1

35. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is **[2002]**

(a) 1 (b) –1

(c) zero (d) does not exist

## TOPIC 2 Limits Using L-hospital's Rule, Evaluation of Limits of the form $1^\infty$, Limits by Expansion Method

36. $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}$ $(a\neq 0)$ is equal to : **[Sep. 03, 2020 (II)]**

(a) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$ (b) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

(c) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (d) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

37. If $\lim_{x\to 1}\frac{x+x^2+x^3+...+x^n-n}{x-1} = 820, (n\in \mathbf{N})$ then the value of $n$ is equal to ________. **[NA Sep. 02, 2020 (I)]**

38. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to : **[Sep. 02, 2020 (II)]**

(a) $e$ (b) 2 (c) 1 (d) $e^2$

39. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ is equal to: **[Jan. 8, 2020 (I)]**

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ (c) $e^2$ (d) e

40. $\lim_{x\to 0}\int_0^x\frac{t\sin(10t)dt}{x}$ is equal to: **[Jan. 8, 2020 (II)]**

(a) 0 (b) $\frac{1}{10}$ (c) $-\frac{1}{5}$ (d) $-\frac{1}{10}$

41. If $\alpha$ and $\beta$ are the roots of the equation $375x^2-25x-2=0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r+\lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to : **[April 12, 2019 (I)]**

(a) $\frac{21}{346}$ (b) $\frac{29}{358}$ (c) $\frac{1}{12}$ (d) $\frac{7}{116}$

**42.** Let $f$ : R $\to$ R be a differentiable function satisfying $f'(3)+f'(2)=0$. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to : **[April 08, 2019 (II)]**

(a) 1 (b) $e^{-1}$ (c) $e$ (d) $e^2$

**43.** For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then **[2018]**

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$$

(a) is equal to 15. (b) is equal to 120.
(c) does not exist (in R). (d) is equal to 0.

**44.** $\lim_{x\to 0}\frac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals. **[Online April 16, 2018]**

(a) $-\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $-\frac{1}{6}$ (d) $\frac{1}{3}$

**45.** Let $p=\lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to : **[2016]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 2 (d) 1

**46.** $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$ is : **[Online April 10, 2016]**

(a) 2 (b) $-\frac{1}{2}$ (c) $-2$ (d) $\frac{1}{2}$

**47.** If $\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, then 'a' is equal to : **[Online April 9, 2016]**

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

**48.** If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of $a$ and $b$, are **[2004]**

(a) $a=1$ and $b=2$ (b) $a=1, b\in R$
(c) $a\in R, b=2$ (d) $a\in R, b\in R$

**49.** Let $f(x)=4$ and $f'(x)=4$. Then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ is given by **[2002]**

(a) 2 (b) $-2$ (c) $-4$ (d) 3

## TOPIC 3 Derivatives of Polynomial & Trigonometric Functions, Derivative of Sum, Difference, Product & Quotient of two functions

**50.** Let $f(x)$ be a polynomial of degree 4 having extreme values at $x=1$ and $x=2$. If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right)=3$ then $f(-1)$ is equal to **[Online April 15, 2018]**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{9}{2}$

**51.** Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then f(2) is equal to : **[2015]**

(a) 0 (b) 4
(c) $-8$ (d) $-4$

**52.** Let $f(1)=-2$ and $f'(x)\geq 4.2$ for $1\leq x\leq 6$. The possible value of $f(6)$ lies in the interval : **[April 25, 2013]**

(a) [15, 19) (b) $(-\infty, 12)$
(c) [12, 15) (d) $[19, \infty)$

**53.** If $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7$, then $\lim_{\alpha\to 0}\frac{f(1-\alpha)-f(1)}{\alpha^3+3\alpha}$ is **[Online May 19, 2012]**

(a) $-\frac{53}{3}$ (b) $\frac{53}{3}$ (c) $-\frac{55}{3}$ (d) $\frac{55}{3}$

# Hints & Solutions

**1.** **(b)** $x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2, -1 \Rightarrow \alpha = 2$

$$\therefore \lim_{x\to 2^+} \frac{\sqrt{1-\cos(x^2-x-2)}}{x-2}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{2}\left|\sin\left(\frac{x^2-x-2}{2}\right)\right|}{x-2}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{2}\sin\frac{(x^2-x-2)}{2}}{\left(\frac{x^2-x-2}{2}\right)} \times \frac{(x^2-x-2)}{2(x-2)}$$

$$= \frac{1}{\sqrt{2}} \lim_{x\to 2^+} \left(\frac{\sin\left(\frac{x^2-x-2}{2}\right)}{\frac{x^2-x-2}{2}}\right) \times \lim_{x\to 2^+} \frac{(x-2)(x+1)}{(x-2)}$$

$$= \frac{1}{\sqrt{2}} \times 1 \times 3 = \frac{3}{\sqrt{2}}$$

**2.** **(b)** Let $L = \lim_{x\to 0} \frac{x\left(e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1\right)}{\sqrt{1+x^2+x^4}-1}$

$$= \lim_{x\to 0} \frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}} - 1}{\frac{\sqrt{1+x^2+x^4}-1}{x}}$$

Put $\frac{\sqrt{1+x^2+x^4}-1}{x} = t$ when $x \to 0 \Rightarrow t \to 0$

$$\therefore L = \lim_{t\to 0} \frac{e^t - 1}{t} = 1$$

**3.** **(b)** Given $\lim_{x\to 0} \left|\frac{1-x+|x|}{\lambda - x + [x]}\right| = L$

Here, L.H.L. $= \lim_{h\to 0} \left|\frac{1+h+h}{\lambda+h-1}\right| = \left|\frac{1}{\lambda-1}\right|$

R.H.L. $= \lim_{h\to 0} \left|\frac{1-h+h}{\lambda+h+0}\right| = \left|\frac{1}{\lambda}\right|$

Given that limit exists. Hence L.H.L. = R.H.L.

$\Rightarrow |\lambda - 1| = |\lambda|$

$\Rightarrow \lambda = \frac{1}{2}$ and $L = \left|\frac{1}{\lambda}\right| = 2$

**4.** **(8)**

$$\lim_{x\to 0} \frac{\left(1-\cos\frac{x^2}{2}\right)}{x^4} \frac{\left(1-\cos\frac{x^2}{4}\right)}{x^4} = 2^{-k}$$

$$\Rightarrow \lim_{x\to 0} \frac{2\sin^2\frac{x^2}{4}}{\frac{x^4}{16}\times 16} \times \frac{2\sin^2\frac{x^2}{8}}{\frac{x^4}{64}\times 64} = 2^{-k}$$

$$\Rightarrow \frac{4}{16\times 64} = 2^{-8} = 2^{-k} \qquad \left[\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$\therefore k = 8$

**5.** **(36)** Let $3^x = t^2$

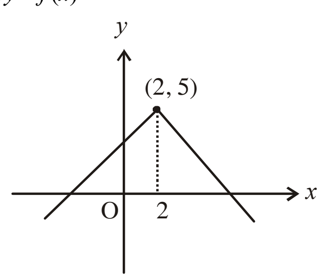
$$\lim_{t\to 3} \frac{t^2 + \frac{27}{t^2} - 12}{\frac{1}{t} - \frac{3}{t^2}} = \lim_{t\to 3} \frac{t^4 - 12t^2 + 27}{t-3}$$

$$= \lim_{t\to 3} \frac{(t^2-3)(t+3)(t-3)}{t-3}$$

$= (3^2 - 3)(3+3) = 36.$

**6.** **(a)** $f(x) = 5 - |x-2|$

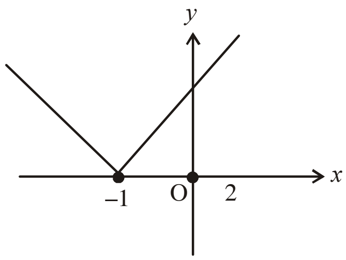
Graph of $y = f(x)$




By the graph $f(x)$ is maximum at $x = 2$

$\therefore \ \alpha = 2 \ g(x) = |x+1|$

Graph of $y = g(x)$




By the graph $g(x)$ is minimum at $x = -1$

$\therefore \quad \beta = -1$

Now, $\lim\limits_{x \to 2} \dfrac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$

$= \lim\limits_{x \to 2} \dfrac{(x-1)(x-3)}{x-4} = \dfrac{1}{2}$

**7.** **(b)** Given limit is,

$$\lim_{x \to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$$

On rationalising,

$$= \lim_{x \to 0} \frac{(x + 2\sin x)\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{(x^2 - \sin^2 x) + (x + 2\sin x)}$$

$$= \lim_{x \to 0} \frac{\left[1 + 2\left(\frac{\sin x}{x}\right)\right]\left[\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right]}{\left(x - \frac{\sin^2 x}{x}\right) + \left(1 + 2\left(\frac{\sin x}{x}\right)\right)}$$

$$= \frac{3 \times 2}{3} = 2 \qquad \left[\because \lim_{x \to 0} \frac{\sin}{x} = 1\right]$$

**8.** **(a)** Given, $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1} = \lim\limits_{x \to K} \left(\dfrac{x^3 - k^3}{x^2 - k^2}\right)$

Taking L.H.S. $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1} \quad \left(\dfrac{0}{0}\text{form}\right)$

$\mathop{\text{Lt}}\limits_{x \to 1} \dfrac{4x^3}{1} = 4$ [Using L Hospital's Rule]

$\therefore \lim\limits_{x \to K} \dfrac{x^3 - k^3}{x^2 - k^2} = 4$

$\Rightarrow \lim\limits_{x \to K} \dfrac{3x^2}{2x} = 4$ [Using L Hospital's Rule]

$\Rightarrow \dfrac{3}{2}k = 4 \Rightarrow k = \dfrac{8}{3}$

**9.** **(c)** $\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{x - 1} = 5$

$\because$ limit is finite. $\therefore 1 - a + b = 0$

$\Rightarrow \lim\limits_{x \to 1} \dfrac{2x - a}{1} = 5 \quad \left(\dfrac{0}{0}\text{form}\right)$ (By L Hospital's rule)

$\Rightarrow 2 - a = 5 \Rightarrow a = -3$ and $b = -4$

Then $a + b = -3 - 4 = -7$

**10.** **(a)** $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$

$$= \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{2\cos^2 \frac{x}{2}}} \qquad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2}\left[1 - \cos\frac{x}{2}\right]} = \lim_{x \to 0} \frac{\sin^2 x}{2\sqrt{2}\sin^2 \frac{x}{4}}$$

$$= \lim_{x \to 0} \frac{\left(\frac{\sin x}{x}\right)^2 .16}{2\sqrt{2}\left(\frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2} = \frac{16}{2\sqrt{2}} = 4\sqrt{2}$$

**11.** **(d)** $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)} = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cot^3 x\left(1 - \frac{\tan x}{\cos^3 x}\right)}{\cos(x + \pi/4)}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 - \tan^4 x)}{\tan^3 x \cos(x + \pi/4)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 + \tan^2 x)(1 - \tan x)(1 + \tan x)}{\tan^3 x\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 + \tan^2 x)(1 + \tan x)(\cos x - \sin x)}{\frac{\sin^3 x}{\cos^2 x}\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)}$$

$$= \frac{(2)(2)}{\frac{1}{(\sqrt{2})(\sqrt{2})}} = 8$$

**12.** **(b)** $\lim\limits_{x \to 1^-} \dfrac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1 - x}} = \lim\limits_{h \to 0} f(1 - h)$

$$= \lim_{h \to 0} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}(1 - h)}}{\sqrt{1 - (1 - h)}}$$

$$= \lim_{h\to 0}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{h}}$$

$$= \lim_{h\to 0}\frac{-\frac{1}{2\sqrt{2\sin^{-1}(1-h)}}\times 2\times\frac{1}{\sqrt{1-(1-h)^2}}(-1)}{\frac{1}{2\sqrt{h}}}$$

$$= \lim_{h\to 0}\frac{\frac{1}{\sqrt{2\sin^{-1}(1-h)}}\frac{1}{\sqrt{h(2-h)}}}{\frac{1}{2\sqrt{h}}}$$

$$=2\times\frac{1}{\sqrt{\pi}}\times\frac{1}{\sqrt{2}}=\sqrt{\frac{2}{\pi}}$$

**13. (a)** RHL is, $\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+(x-0)^2}{x^2}$

$$= \lim_{x\to 0^+}\left(\frac{\tan(\pi\sin^2 x}{x^2}+1\right)=1+\pi$$

And LHL is, $\lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

$$= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+x^2+\sin^2 x-2x\sin x}{x^2}$$

$=\pi+1+1-2=\pi$

Since, LHL $\neq$ RHL

Hence, limit does not exist.

**14. (d)** $\lim_{x\to 0}\frac{x\cot 4x}{\sin^2 x.\cot^2 2x}=\lim_{x\to 0}\frac{x.\tan^2 2x}{\sin^2 x.\tan 4x}$

$$= \lim_{x\to 0}\left(\frac{x}{\sin x}\right)^2.\left(\frac{\tan 2x}{2x}\right)^2.\left(\frac{4x}{\tan 4x}\right).\frac{4}{2^2}=1$$

**15. (b)** $\lim_{x\to 1^+}\frac{(1-|x|+\sin(|1-x|))\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$

$$= \lim_{h\to 0}\frac{(1-|1+h|+\sin(|1-1-h|))\sin\left(\frac{\pi}{2}[1-1-h]\right)}{|1-1-h|[1-1-h]}$$

$$= \lim_{h\to 0}\frac{(1-1-h+\sinh)\sin\left(\frac{\pi}{2}(-1)\right)}{h([0-h])}$$

$$= \lim_{h\to 0}\frac{(-h+\sin h)\sin\left(-\frac{\pi}{2}\right)}{h(-1)}=0$$

**16. (a)** $L=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$=\frac{1}{2\sqrt{2}\times 2}=\frac{1}{4\sqrt{2}}$$

**17. (a)** $\lim_{x\to 0^-}\frac{x([x]+|x|)\cdot\sin[x]}{|x|}$

$$= \lim_{h\to 0}\frac{(0-h)([0-h]+|0-h|)\cdot\sin[0-h]}{|0-h|}$$

$$= \lim_{h\to 0}\frac{(-h)(-1+h)\sin(-1)}{h}$$

$$= \lim_{h\to 0}(1-h)\sin(-1)=-\sin 1$$

**18. (d)** Let, $L=\lim_{x\to 0}\frac{(x\tan 2x-2x\tan x)}{(1-\cos 2x)^2}=\lim_{x\to 0}K$ (say)

$$\Rightarrow K=\frac{x\left[\frac{2\tan x}{1-(\tan x)^2}\right]-2x\tan x}{(1-(1-2\sin^2 x))^2}$$

$$=\frac{2x\tan x-[2x\tan x-2x\tan^3 x]}{4\sin^4 x\times(1-\tan^2 x)}$$

$$=\frac{2x\tan^3 x}{4\sin^4 x\times(1-\tan^2 x)}=\frac{2x\tan^3 x}{4\sin^4 x\times\left(\frac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)}$$

$$=\frac{2x\frac{\sin^3 x}{\cos^3 x}}{4\sin^4 x\times\left(\frac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)}$$

$$\Rightarrow K=\frac{x}{2\sin x\times(\cos^2 x-\sin^2 x)\cos x}$$

$$\therefore L=\lim_{x\to 0}\frac{x}{2\sin x}\times\lim_{x\to 0}\frac{1}{\cos x(\cos^2 x-\sin^2 x)}$$

$$=\lim_{x\to 0}\frac{x}{2\sin x}\times\lim_{x\to 0}\frac{1}{\cos 0(\cos^2 0-\sin^2 0)}=\frac{1}{2}$$

**19.** **(c)** $\lim_{x\to\frac{\pi}{2}}\frac{\cot x(1-\sin x)}{-8\left(x-\frac{\pi}{2}\right)^3}=\lim_{x\to\frac{\pi}{2}}\frac{\cot x(1-\sin x)}{8\left(\frac{\pi}{2}-x\right)^3}$

Put $\frac{\pi}{2}-x=t\Rightarrow$ as $x\to\frac{\pi}{2}\Rightarrow t\to 0$

$$=\lim_{t\to 0}\frac{\cot\left(\frac{\pi}{2}-t\right)\left(1-\sin\left(\frac{\pi}{2}-t\right)\right)}{8t^3}$$

$$=\lim_{t\to 0}\frac{\tan t(1-\cos t)}{8t^3}=\lim_{t\to 0}\frac{\tan t}{8t}.\frac{1-\cos t}{t^2}$$

$$=\frac{1}{8}.1.\frac{1}{2}=\frac{1}{16}$$

**20.** **(b)** Let $A=\lim_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

Rationalise

$$\Rightarrow A=\lim_{x\to 3}\frac{(3x-9)\times\left(2x-4+\sqrt{2}\right)}{\{(2x-4)-2\}\times\left(\sqrt{3x}+3\right)}$$

$$=\lim_{x\to 3}\frac{3(x-3)}{2(x-3)}\times\frac{\sqrt{2x-4}+\sqrt{2}}{\left(\sqrt{3x}+3\right)}=\frac{3}{2}\times\frac{2\sqrt{2}}{6}=\frac{1}{\sqrt{2}}$$

**21.** **(a)** Multiply and divide by $x$ in the given expression, we get

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}\frac{(3+\cos x)}{1}\cdot\frac{x}{\tan 4x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}\cdot\frac{3+\cos x}{1}\cdot\frac{x}{\tan 4x}$$

$$=2\lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\lim_{x\to 0}3+\cos x\cdot\lim_{x\to 0}\frac{x}{\tan 4x}$$

$$=2.4\,\frac{1}{4}\lim_{x\to 0}\frac{4x}{\tan 4x}=2.4.\frac{1}{4}=2$$

**22.** **(c)** $\lim_{x\to 0}\frac{2xe^{x^2}+\sin x}{2\sin x\cos x}$

$$\lim_{x\to 0}\left(\frac{x}{\sin x}e^{x^2}+\frac{1}{2}\right)\frac{1}{\cos x}=1+\frac{1}{2}=\frac{3}{2}$$

**23.** **(b)** $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$$=\lim_{x\to 0}\frac{\sin\left[\pi(1-\sin^2 x)\right]}{x^2}$$

$$=\lim_{x\to 0}\sin\frac{(\pi-\pi\sin^2 x)}{x^2}\qquad[\because\ \sin(\pi-\theta)=\sin\theta]$$

$$=\lim_{x\to 0}\sin\frac{(\pi\sin^2 x)}{\pi\sin^2 x}\times\frac{\pi\sin^2 x}{x^2}$$

$$=\lim_{x\to 0}1\times\pi\left(\frac{\sin x}{x}\right)^2=\pi$$

**24.** **(d)** $\lim_{x\to 2}\frac{\tan(x-2)\{x^2+(k-2)x-2k\}}{x^2-4x+4}=5$

$$\Rightarrow\lim_{x\to 2}\frac{\tan(x-2)\{x^2+kx-2x-2k\}}{(x-2)^2}=5$$

$$\Rightarrow\lim_{x\to 2}\frac{\tan(x-2)\{x(x-2)+k(x-2)\}}{(x-2)\times(x-2)}=5$$

$$\Rightarrow\lim_{x\to 2}\left(\frac{\tan(x-2)}{(x-2)}\right)\times\lim_{x\to 2}\left(\frac{(k+x)\cancel{(x-2)}}{\cancel{(x-2)}}\right)=5$$

$$\Rightarrow 1\times\lim_{x\to 2}(k+x)=5\qquad\left\{\because\lim_{h\to 0}\frac{\tan h}{h}=1\right\}$$

or $k+2=5$

$\Rightarrow$ $\boxed{k=3}$

**25.** **(d)** Multiply and divide by $x$ in the given expression, we get

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}\frac{(3+\cos x)}{1}\cdot\frac{x}{\tan 4x}$$

$$\left[\because 1-\cos 2x=2\sin^2\frac{x}{2}\right]$$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}\cdot\frac{3+\cos x}{1}\cdot\frac{x}{\tan 4x}$$

$$=2\lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\lim_{x\to 0}(3+\cos x)\cdot\lim_{x\to 0}\frac{4x}{\tan 4x}\times\frac{1}{4}$$

$$=2.4.\frac{1}{4}=2$$

**26.** **(d)** Consider, $\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$

$$=\lim_{x\to 0}\frac{\sin\left(\pi-\pi\sin^2 x\right)}{x^2}\qquad[\because\sin(\pi-\theta)=\sin\theta]$$

$$=\lim_{x\to 0}\frac{\sin\left(\pi\sin^2 x\right)}{\pi\sin^2 x}\times\frac{\left(\pi\sin^2 x\right)}{x^2}=\pi$$

**27.** **(b)** Consider $\lim_{x\to 0}\left(\frac{x-\sin x}{x}\right)\sin\left(\frac{1}{x}\right)$

$= \lim_{x\to 0}\left[\frac{x\left(1-\frac{\sin x}{x}\right)}{x}\right]\times \lim_{x\to 0}\sin\left(\frac{1}{x}\right)$

$= \lim_{x\to 0}\left[1-\frac{\sin x}{x}\right]\times \lim_{x\to 0}\sin\left(\frac{1}{x}\right)$

$= \left[1-\lim_{x\to 0}\frac{\sin x}{x}\right]\times \lim_{x\to 0}\sin\left(\frac{1}{x}\right)$

$= 0\times \lim_{x\to 0}\sin\left(\frac{1}{x}\right) = 0$

**28.** **(d)** Given that $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$

$\Rightarrow \lim_{x\to 5}[(f(x))^2-9]=0$

$\Rightarrow \left[\lim_{x\to 5}f(x)\right]^2 = 9 \Rightarrow \lim_{x\to 5}f(x)=3$

**29.** **(d)** $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ $\left[\because 1-\cos\theta = 2\sin^2\frac{v}{2}\right]$

$= \lim_{x\to 2}\frac{\sqrt{2}\,|\sin(x-2)|}{x-2}$

$\underset{(\text{at } x=2)}{\text{L.H.L}} = -\lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)} = -\sqrt{2}$

$\underset{(\text{at } x=2)}{\text{R.H.L}} = \lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{(x-2)} = \sqrt{2}$

Thus $\underset{(\text{at } x=2)}{\text{L.H.L}} \neq \underset{(\text{at } x=2)}{\text{R.H.L}}$

Hence, $\lim_{x\to 2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ does not exist.

**30.** **(d)** Given that f(x) is a positive increasing function.

$\therefore\ 0 < f(x) < f(2x) < f(3x)$

Divided by $f(x)$

$\Rightarrow 0 < 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$

$\Rightarrow \lim_{x\to\infty} 1 \le \lim_{x\to\infty}\frac{f(2x)}{f(x)} \le \lim_{x\to\infty}\frac{f(3x)}{f(x)}$

By Sandwich Theorem.

$\Rightarrow \lim_{x\to\infty}\frac{f(2x)}{f(x)} = 1$

**31.** **(a)** Given that

$ax^2+bx+c = a(x-\alpha)(x-\beta)$

$\lim_{x\to\alpha}\frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2}$

$= \lim_{x\to\alpha}\frac{2\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$

$= \lim_{x\to\alpha}\frac{2}{(x-\alpha)^2}\times\frac{\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}}\times\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}$

$= \frac{a^2(\alpha-\beta)^2}{2}.$

**32.** **(d)** $\lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right).(1-\sin x)}{(\pi-2x)^3}$

Let $x=\frac{\pi}{2}+y; y\to 0$

$= \lim_{y\to 0}\frac{\tan\left(-\frac{y}{2}\right).(1-\cos y)}{(-2y)^3}$

$= \lim_{y\to 0}\frac{-\tan\frac{y}{2}\,2\sin^2\frac{y}{2}}{(-8).\frac{y^3}{8}.8}$ $\left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$

$= \lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left[\frac{\sin y/2}{y/2}\right]^2 = \frac{1}{32}$

$\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = \lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1\right]$

**33.** **(d)** Since, $\lim_{x\to 0^-}[x] = -1 \neq \lim_{x\to 0^+}[x] = 0$. So $\lim_{x\to 0}[x]$ does not exist, hence the required limit does not exist.

**34.** **(a)** $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x = \lim_{x\to\infty}\left(1+\frac{4x+1}{x^2+x+2}\right)^x$

$= \lim_{x\to\infty}\left[\left(1+\frac{4x+1}{x^2+x+2}\right)^{\frac{x^2+x+2}{4x+1}}\right]^{\frac{(4x+1)x}{x^2+x+2}}$

$= e^{\lim_{x\to\infty}\frac{4x^2+x}{x^2+x+2}}$ $\left[\because \lim_{x\to\infty}(1+\lambda x)^{\frac{1}{x}} = e^{\lambda}\right]$

$= e^{\lim_{x\to\infty}\frac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{2}{x^2}}} = e^4$ $\left[\because \frac{1}{\infty} = 0\right]$

35. (d) $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$;

$\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}\Rightarrow \lim_{x\to 0}\frac{|\sin x|}{x}$ $\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$

The limit of above does not exist as

LHS $=-1\neq$ RHL $=1$

36. (b) $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}$ $\left[\frac{0}{0}\text{ case}\right]$

Apply L'Hospital rule

$$\lim_{x\to a}\frac{\frac{1}{3}(a+2x)^{-2/3}\cdot 2-\frac{1}{3}\cdot(3x)^{-2/3}\cdot 3}{\frac{1}{3}(3a+x)^{-2/3}\cdot -\frac{1}{3}(4x)^{-2/3}\cdot 4}$$

$$=\frac{\frac{1}{3}(3a)^{-2/3}\cdot(2-3)}{\frac{1}{3}(4a)^{-2/3}\cdot(1-4)}=\frac{3^{-2/3}}{4^{-2/3}}\cdot\frac{1}{3}=\frac{2^{4/3}}{9^{1/3}}\cdot\frac{1}{3}=\frac{2}{3}\cdot\left(\frac{2}{9}\right)^{1/3}$$

37. (40)

$$\lim_{x\to 1}\frac{x+x^2+x^3+....+x^n-n}{x-1}=820\left(\frac{0}{0}\text{ case}\right)$$

$$\lim_{x\to 1}\frac{1+2x+3x^2+....+nx^{n-1}}{1}=820$$

(Using L' Hospital rule)

$\Rightarrow 1+2+3+....+n=820$

$\Rightarrow \frac{n(n+1)}{2}=820$ $\Rightarrow n^2+n-1640=0$

$\Rightarrow n=40,\ n\in \mathrm{N}$

38. (d) $\lim_{x\to 0}\left(\frac{1+\tan x}{1-\tan x}\right)^{1/x}$

$\Rightarrow e^{\lim_{x\to 0}\frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}$ $\Rightarrow e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{1+\tan x}{1-\tan x}-1\right)}$

$\Rightarrow e^{\lim_{x\to 0}\left(\frac{2\tan x}{1-\tan x}\right)\frac{1}{x}}=e^{\lim_{x\to 0}\left(\frac{\tan x}{x}\right)\left(\frac{2}{1-\tan x}\right)}=e^2$

39. (b) Let R $=\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}=e^{\lim_{x\to 0}\frac{1}{x^2}\left\{\frac{3x^2+2}{7x^2+2}-1\right\}}$

$=e^{\lim_{x\to 0}\frac{1}{x^2}\left\{\frac{-4x^2}{7x^2+2}\right\}}=e^{\frac{-4}{2}}=e^{-2}=\frac{1}{e^2}$

40. (a) Using L' Hospital rule,

$$\lim_{x\to 0}\frac{x\sin(10x)}{1}=0$$

41. (c) Given equation is, $375x^2-25x-2=0$

Sum and product of the roots are,

$\alpha+\beta=\frac{25}{375}$ and $\alpha\beta=\frac{-2}{375}$

$\lim_{n\to\infty}\sum_{r=1}^{n}(\alpha^r+\beta^r)$

$=(\alpha+\alpha^2+\alpha^3+...\infty(\beta+\beta^2+\beta^3...+\infty)$

$=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}=\frac{\alpha+\beta-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$

$=\frac{\frac{25}{375}+\frac{4}{375}}{1-\frac{25}{375}-\frac{2}{375}}=\frac{29}{375-25-2}=\frac{29}{348}=\frac{1}{12}$

42. (a) $I=\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ $[1^\infty$ form$]$

$\Rightarrow I=e^{I_1}$, where

$I_1=\lim_{x\to 0}\left(\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right)\right)\left(\frac{1}{x}\right)$

$=\lim_{x\to 0}\left(\frac{1}{x}\right)\left(\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)$

$\left(\frac{0}{0}\text{ form}\right)$

By L. Hospital Rule,

$I_1=\lim_{x\to 0}\left(\frac{f'(3+x)+f'(2-x)}{1}\right)\lim_{x\to 0}\left(\frac{1}{1+f(2-x)-f(2)}\right)$

$=f'(3)+f'(2)=0$

$\Rightarrow I=e^{I_1}=e^0=1$

43. (b) Since, $\lim_{x\to 0^+}x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+...+\left[\frac{15}{x}\right]\right)$

$=\lim_{x\to 0^+}x\left(\frac{1+2+3+...+15}{x}\right)-\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+...+\left\{\frac{15}{x}\right\}\right)$

$\because 0\le\left\{\frac{r}{x}\right\}<1 \Rightarrow 0\le x\left\{\frac{r}{x}\right\}<x$

$\therefore \lim_{x\to 0^+}x\left(\frac{1+2+3+...+15}{x}\right)=\frac{15\times 16}{2}=120$

**44.** **(c)** Let $L = \lim_{x \to 0} \frac{(27+x)^{\frac{1}{3}} - 3}{9-(27+x)^{\frac{2}{3}}}$

Here 'L' is in the indeterminate form i.e., $\frac{0}{0}$

$\therefore$ using the L'Hospital rule we get:

$$L = \lim_{x \to 0} \frac{\frac{1}{3}(27+x)^{\frac{-2}{3}}}{-\frac{2}{3}(27+x)^{\frac{-1}{3}}} = \frac{\frac{1}{3} \times (27)^{\frac{-2}{3}}}{\frac{-2}{3} \times 27^{\frac{-1}{3}}} = -\frac{1}{6}$$

**45.** **(a)** $\ln p = \lim_{x \to 0^+} \frac{1}{2x} \ln(1+\tan^2 \sqrt{x})$

$$\lim_{x \to 0^+} \frac{1}{x} \ln\left(\sec \sqrt{x}\right)$$

Applying L hospital's rule :

$$= \lim_{x \to 0^+} \frac{\sec \sqrt{x} \tan \sqrt{x}}{\sec \sqrt{x} \cdot 2\sqrt{x}} = \lim_{x \to 0^+} \frac{\tan \sqrt{x}}{2\sqrt{x}} = \frac{1}{2}$$

**46.** **(c)** $\lim_{x \to 0} \frac{(1-\cos 2x)^2}{2x \tan x - x \tan 2x}$

$$= \lim_{x \to 0} \frac{(2\sin^2 x)^2}{2x\left(x + \frac{x^3}{3} + \frac{2x^5}{15} + .....\right) - x\left(2x + \frac{2^3 x^3}{3} + 2\frac{2^5 x^5}{15} + .....\right)}$$

$$= \lim_{x \to 0} \frac{4\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - .....\right)^4}{x^4\left(\frac{2}{3} - \frac{8}{5}\right) + x^6\left(\frac{4}{15} - \frac{64}{15}\right)}$$

$$= \lim_{x \to 0} \frac{4\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - .....\right)^4}{-2 + x^2\left(-\frac{60}{15}\right) + .....}$$

(dividing numerator & denominator by $x^4$)

$= -2$

**47.** **(b)** $\lim_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x}$ ($1^\infty$ form)

$$= e\left[\lim_{x \to \infty} \left(1 + \frac{a}{x} - \frac{4}{x^2} - 1\right) 2x\right]$$

$$= e \lim_{x \to \infty} \left(2a - \frac{8}{x}\right) = e^{2a}$$

$\therefore \quad 2a = 3 \Rightarrow a = 3/2$

**48.** **(b)** We know that $\lim_{x \to \infty} (1+x)^{\frac{1}{x}} = e$

Given that $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$

$$\Rightarrow \lim_{x \to \infty} \left[\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{\left(\frac{1}{\frac{a}{x} + \frac{b}{x^2}}\right)}\right]^{2x\left(\frac{a}{x} + \frac{b}{x^2}\right)} = e^2$$

$$\Rightarrow e^{\lim_{x \to \infty} 2\left[a + \frac{b}{x}\right]} = e^2 \Rightarrow e^{2a} = e^2$$

$\Rightarrow a = 1$ and $b \in R$

**49.** **(c)** Given that $f(2) = 4$ and $f(2) = 4$

We have, $\lim_{x \to 2} \frac{xf(2) - 2f(x)}{x-2}$, $\left(\frac{0}{0}\right)$

Applying L-Hospital's rule, we get

$$= \lim_{x \to 2} f(2) - 2f'(x) = f(2) - 2f'(2)$$

$= 4 - 2 \times 4 = -4.$

**50.** **(d)** $\because f(x)$ has extremum values at $x = 1$ and $x = 2$

$\because f'(1) = 0$ and $f'(2) = 0$

As, $f(x)$ is a polynomial of degree 4.

Suppose $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

$$\because \lim_{x \to 0} \left(\frac{f(x)}{x^2} + 1\right) = 3$$

$$\Rightarrow \lim_{x \to 0} \left(\frac{Ax^4 + Bx^3 + Cx^2 + Dx + E}{x^2} + 1\right) = 3$$

$$\Rightarrow \lim_{x \to 0} \left(Ax^2 + Bx + C + \frac{D}{x} + \frac{E}{x^2} + 1\right) = 3$$

As limit has finite value, so $D = 0$ and $E = 0$

Now $A(0)^2 + B(0) + C + 0 + 0 + 1 = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$

$f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$f'(1) = 0 \Rightarrow 4A(1) + 3B(1) + 2C(1) + D = 0$

$\Rightarrow 4A + 3B = -4$ ...(i)

$f'(2) = 0 \Rightarrow 4A(8) + 3B(4) + 2C(2) + D = 0$

$\Rightarrow 8A + 3B = -2$ ...(ii)

From equations (i) and (ii), we get

$A = \frac{1}{2}$ and $B = -2$

So, $f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

Therefore, $f(-1) = \frac{(-1)^4}{2} - 2(-1)^3 + 2(-1)^2$

$= \frac{1}{2} + 2 + 2 = \frac{9}{2}$. Hence $f(-1) = \frac{9}{2}$

**51.** **(a)** $\lim_{x\to0}\left[1+\frac{f(x)}{x^2}\right]=3$

$\Rightarrow \lim_{x\to0}\frac{f(x)}{x^2}=2$

So, f(x) contain terms in $x^2$, $x^3$ and $x^4$.

Let $f(x)=a_1x^2+a_2x^3+a_3x^4$

Since $\lim_{x\to0}\frac{f(x)}{x^2}=2 \Rightarrow a_1=2$

Hence, $f(x)=2x^2+a_2x^3+a_3x^4$

$f'(x)=4x+3a_2x^2+4a_3x^3$

As given : $f'(1)=0$ adn $f'(2)=0$

Hence, $4+3a_2+4a_3=0$ ...(i)

and $8+12a_2+32a_3=0$ ...(ii)

By 4x(i) – (ii), we get

$16+12a_2+16a_3-(8+12a_2+32a_3)=0$

$\Rightarrow 8-16a_3=0 \Rightarrow a_3=1/2$

and by eqn. (i), $4+3a_2+4/2=0 \Rightarrow a_2=-2$

$\Rightarrow f(x)=2x^2-2x^3+\frac{1}{2}x^4$

$f(2)=2\times4-2\times8+\frac{1}{2}\times16=0$

**52.** **(d)** Given $f(1)=-2$ and $f'(x)\ge4.2$ for $1\le x\le6$

Consider $f'(x)=\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f(x+h)-f(x)=f'(x).h\ge(4.2)h$

So, $f(x+h)\ge f(x)+(4.2)h$

put $x=1$ and $h=5$, we get

$f(6)\ge f(1)+5(4.2) \Rightarrow f(6)\ge19$

Hence $f(6)$ lies in $[19, \infty)$

**53.** **(b)** Let $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7$

$f'(x)=30x^9-56x^7+30x^5-63x^2+6x$

$f'(1)=30-56+30-63+6$

$=66-63-56=-53$

Consider $\lim_{\alpha\to0}\frac{f(1-\alpha)-f(1)}{\alpha^3+3\alpha}$

$=\lim_{\alpha\to0}\frac{f'(1-\alpha)(-1)-0}{3\alpha^2+3}$ (By using L'hospital rule)

$=\frac{f'(1-0)(-1)}{3(0)^2+3}=\frac{-f'(1)}{3}=\frac{53}{3}$